

# Semester One Examination, 2021 **Question/Answer booklet**

# **MATHEMATICS SPECIALIST** UN

Se	ction	One:	
Ca	<b>Icula</b>	tor-free	3

UNIT 1		•	, ,	dministrator,   n label in this	
Section One: Calculator-free					)
WA student number:	In figures				
	In words				
	Your name	2			
Time allowed for this seeding time before commend Working time:	cing work: f	ive minutes		 of additional ooklets used able):	

# Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet Formula sheet

### To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,

correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100 92		65
				Total	100

#### Instructions to candidates

- 1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen.
  Do not use erasable or gel pens.
- 3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- 4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- 5. It is recommended that you do not use pencil, except in diagrams.
- 6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only					
Question	Maximum	Mark			
1	6				
2	5				
3	6				
4	4				
5	6				
6	8				
7	8				
8	7				
S1 Total	50				
S1 Wt (×0.7)	35%				
S2 Wt	65%				
Total	100%				

**Section One: Calculator-free** 

35% (50 Marks)

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

Points P, Q, R and S lie on the circumference of a circle so that the chords PR and QS intersect at point T.

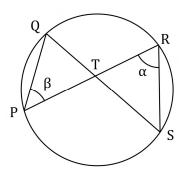
(a) Sketch a diagram to show triangle PST and triangle QTR and prove that they are similar. (4 marks)

(b) In the case when the lengths of PT, QT and RT are 5 cm, 8 cm and 12 cm respectively, determine the length of ST. (2 marks)

Question 2 (5 marks)

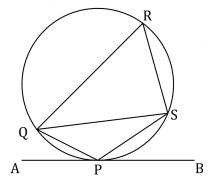
(a) In the circle shown, chords PR and QS intersect at T,  $\angle PQT = 64^{\circ}$  and  $\angle PTS = 115^{\circ}$ .

Determine the size of angles  $\alpha$  and  $\beta$ . (2 marks)



(b) In the circle below, PQRS is a cyclic quadrilateral and AB is a tangent to the circle at P.

Given that  $\angle SPB = 33^{\circ}$  and  $\angle QSP = 25^{\circ}$ , determine the size of  $\angle QRS$ . (3 marks)



DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

**Question 3** (6 marks)

Let  $\mathbf{a} = 3\mathbf{i} - 6\mathbf{j}$  and  $\mathbf{b} = -\mathbf{i} + \mathbf{j}$ .

(a) Determine  $|2\mathbf{a} + 3\mathbf{b}|$ . (3 marks)

Determine the vectors  $\mathbf{m}$  and  $\mathbf{n}$  given that  $2\mathbf{a} + \mathbf{b} = \mathbf{m} - \mathbf{n}$  and  $2\mathbf{b} - \mathbf{a} = 2\mathbf{m} + \mathbf{n}$ . (b)

(3 marks)

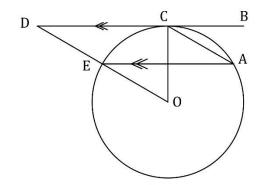
(4 marks)

#### **Question 4**

In the diagram, BD is a tangent at C to the circle with centre O, OD intersects the circle at E and chord AE is parallel to tangent BD.

Determine the size of  $\angle ACB$  when when the size of  $\angle CDE = 36^{\circ}$ .

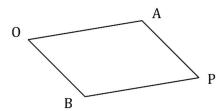
Justify your answer.



Question 5 (6 marks)

The diagram shows parallelogram OAPB.

A theorem states that the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of its sides.



(a) Complete the following expression of the theorem using vector notation:

(1 mark)

$$\left|\overrightarrow{OP}\right|^2 + \left|\overrightarrow{BA}\right|^2 =$$

(b) Letting  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ , use a vector method to prove the theorem. (5 marks)

Question 6 (8 marks)

(a) Determine the vector(s) that are parallel to 3i - 4j and have the same magnitude as i + 7j. (4 marks)

(b) Two vectors are  $4\mathbf{i} + (1 - \mu)\mathbf{j}$  and  $\mathbf{i} + (2\mu + 5)\mathbf{j}$ , where  $\mu$  is a constant.

Determine the value(s) of  $\mu$  so that the vectors are perpendicular.

(4 marks)

Question 7

(8 marks)

(a) Show that  $4\binom{5}{1} = 2\binom{5}{2}$ .

(2 marks)

(b) Show that  $(n-r)\binom{n}{r} = (r+1)\binom{n}{r+1}$  for all n and r that are positive integers, n > r. (4 marks)

(c) Hence, or otherwise, evaluate  $\binom{65}{59}$ , given that  $\binom{65}{60} = 8259888$ . (2 marks)

Question 8 (7 marks)

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Points P, Q and R have position vectors (0,5), (-6,-5) and (10,-1) respectively.

(a) Determine the position vector of M, the midpoint of Q and R.

(1 mark)

(b) Determine the vectors  $\overrightarrow{QR}$  and  $\overrightarrow{PM}$ .

(1 mark)

(c) Show that  $\overrightarrow{QR}$  and  $\overrightarrow{PM}$  are perpendicular.

(2 marks)

(d) Hence, or otherwise, determine the area of triangle PQR.

(3 marks)

Supplementary page

Question number: \_\_\_\_\_